Straight Line

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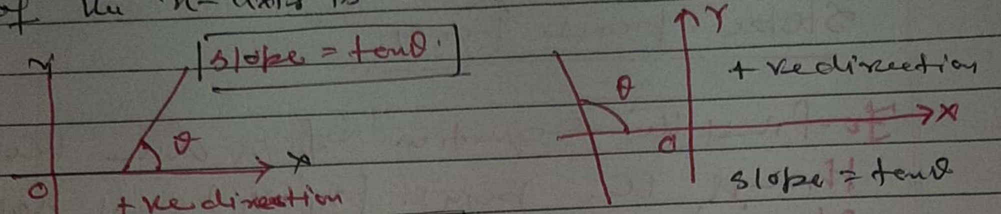
Slope and intercept of St. line

Slope of a line (Gradient):- The tangent of the angle that a line makes with the positive direction of the x-axis is called the slope of the line.



$\boxed{\text{slope} = \tan\theta}$

$\boxed{\text{It is denoted by } m}$

The slope of a line passing through points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by $\boxed{m = \tan\theta = \frac{y_2 - y_1}{x_2 - x_1}}$

Angle between two lines

The angle θ between the two lines having slopes m_1 and m_2 is given by $\boxed{\tan\theta = \pm\frac{(m_1 - m_2)}{1 + m_1 m_2}}$

If we take the acute angle between two lines, then $\boxed{\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|}$

If the lines are parallel, $\boxed{\text{then } m_1 = m_2}$

If the lines are perpendicular, $\boxed{\text{then } m_1 m_2 = -1}$

Collinearity of Three points:-

If three points $P(h, k)$, $Q(x_1, y_1)$ and $R(x_2, y_2)$ are such that slope of PQ = slope of QR,

i.e. $\frac{y_1 - k}{x_1 - h} = \frac{y_2 - y_1}{x_2 - x_1}$

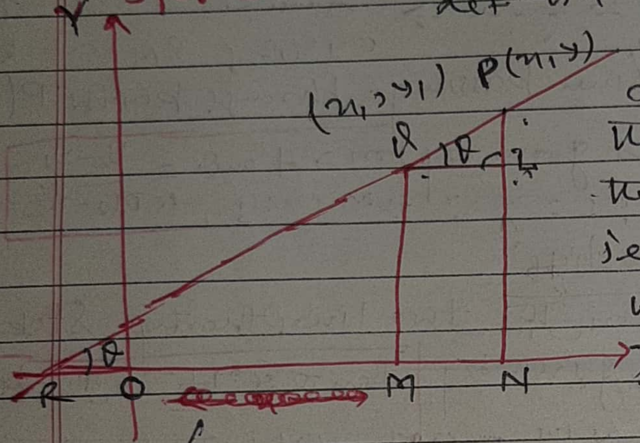
or $(h - x_1)(y_2 - y_1) = (k - y_1)(x_2 - x_1)$ then they are said to be collinear.

Standard equation of St. Line

There are many Standard forms of a Line.
They are as follows:-

Slope-point form

To find the equation of a line which passes through the point (x_1, y_1) and has the slope 'm'. Let $Q(x_1, y_1)$ be the point through which the line passes and its inclination with the positive direction of the x-axis be θ.
i.e. $m = \tan\theta$.
We have to find the eqn of the line.



Let $P(x, y)$ be any point on the line draw $QM \perp OX$, $PN \perp OX$ and $QL \perp PN$.
From the defn of Co-ordinates.

$OM = x$, $QM = y$, $ON = x$ & $PN = y$.

Clearly $QL = MN = ON - OM = x - x_1$

$PL = PN - LN = PN - QM = y - y_1$

Also $\angle PQL = \theta$. [for $QL \parallel OX$]

From the right angle Δ PLQ

$$\tan\angle PQL = \frac{PL}{QL} \Rightarrow \tan\theta = \frac{y - y_1}{x - x_1}$$

$$\Rightarrow m = \frac{y - y_1}{x - x_1} \Rightarrow \boxed{y - y_1 = m(x - x_1)}$$

This is eqn of slope point form

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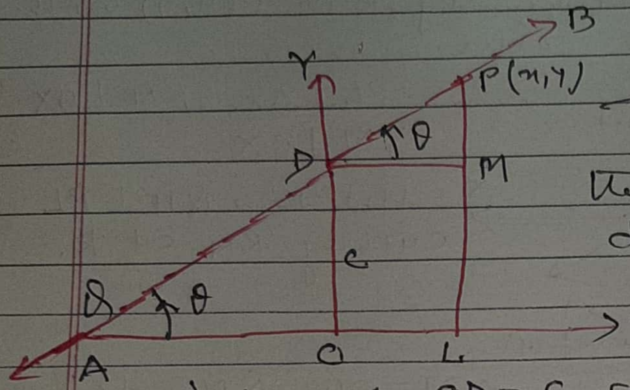
Slope - Intercept form

To find the equation of a straight line which has the slope 'm' and cuts off an intercept c from the y-axis [Passes through the point (0, c)]



Let AB be the line whose slope is $m = \tan\theta$, where θ is the inclination of the line with the positive direction of the x-axis.

Let it cuts off an intercept $OD = c$ on the y-axis.

Let $P(x, y)$ be any point on the line.

draw $PL \perp OX$, $DM \perp PL$

By defn of Co-ordinates

$OL = x$ and $PL = y$

As $DM \parallel QL \therefore \angle PDM = \angle PQL$

But $\angle PQL = \theta$. $\therefore \angle PDM = \theta$

Now $PM = PL - ML = PL - DO = y - c$

and $DM = OL = x$.

From right angle $\triangle PDM$

$$\tan\theta = \frac{PM}{DM} = \frac{y-c}{x}$$

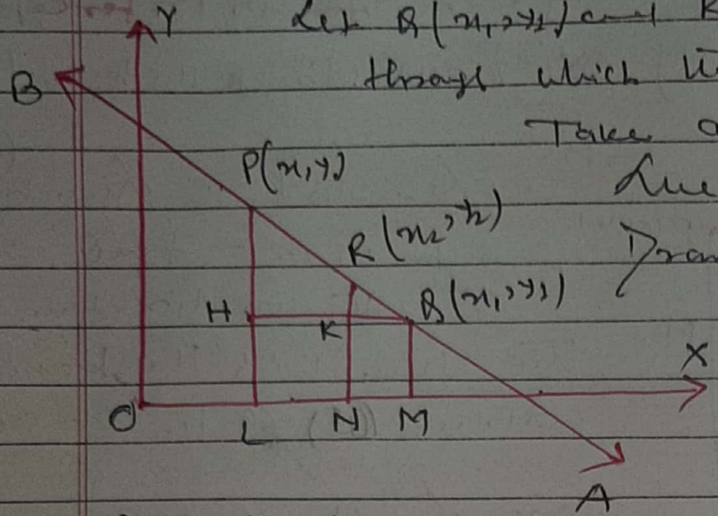
$$\Rightarrow m = \frac{y-c}{x} \Rightarrow y - c = mx$$

$$\Rightarrow \boxed{y = mx + c}$$

This is required eqn of the line

Two point form

To find the equation of a line passing through the two points (x_1, y_1) and (x_2, y_2)



Let $Q(x_1, y_1)$ and $R(x_2, y_2)$ be the given points through which the line AB passes.

Take any point $P(x, y)$ on the line.

Draw $PL \perp OX$, $RN \perp OX$, $QM \perp OX$

Also draw $QH \perp PL$ cutting RN at K.

By defn of co-ordinates)-

$OL = x,\ PL = y,\ OM = x_1,\ QM = y_1$

$ON = x_2,\ RN = y_2$

Now, $HQ = LM = OM - OL = x_1 - x$

$PH = PL - LH = PL - QM = y - y_1$

$KQ = NM = OM - ON = x_1 - x_2$

$RK = RN - KN = RN - QM = y_2 - y_1$

From the similar triangles PHQ and RKQ

$$\frac{PH}{RK} = \frac{HQ}{KQ} \Rightarrow \frac{y - y_1}{y_2 - y_1} = \frac{x_1 - x}{x_1 - x_2}$$

$$\Rightarrow \frac{y - y_1}{-(y_1 - y_2)} = \frac{-(x - x_1)}{x_1 - x_2} \Rightarrow \frac{y - y_1}{y_1 - y_2} = \frac{x - x_1}{x_1 - x_2}$$

$$\Rightarrow \frac{y - y_1}{-(y_1 - y_2)} = \frac{-(x - x_1)}{x_1 - x_2}$$

$$\Rightarrow \frac{y - y_1}{y_1 - y_2} = \frac{x - x_1}{x_1 - x_2}$$

$$\Rightarrow \boxed{y - y_1 = \frac{y_1 - y_2}{x_1 - x_2}(x - x_1)}$$

This is the required equation of the line in two point form.